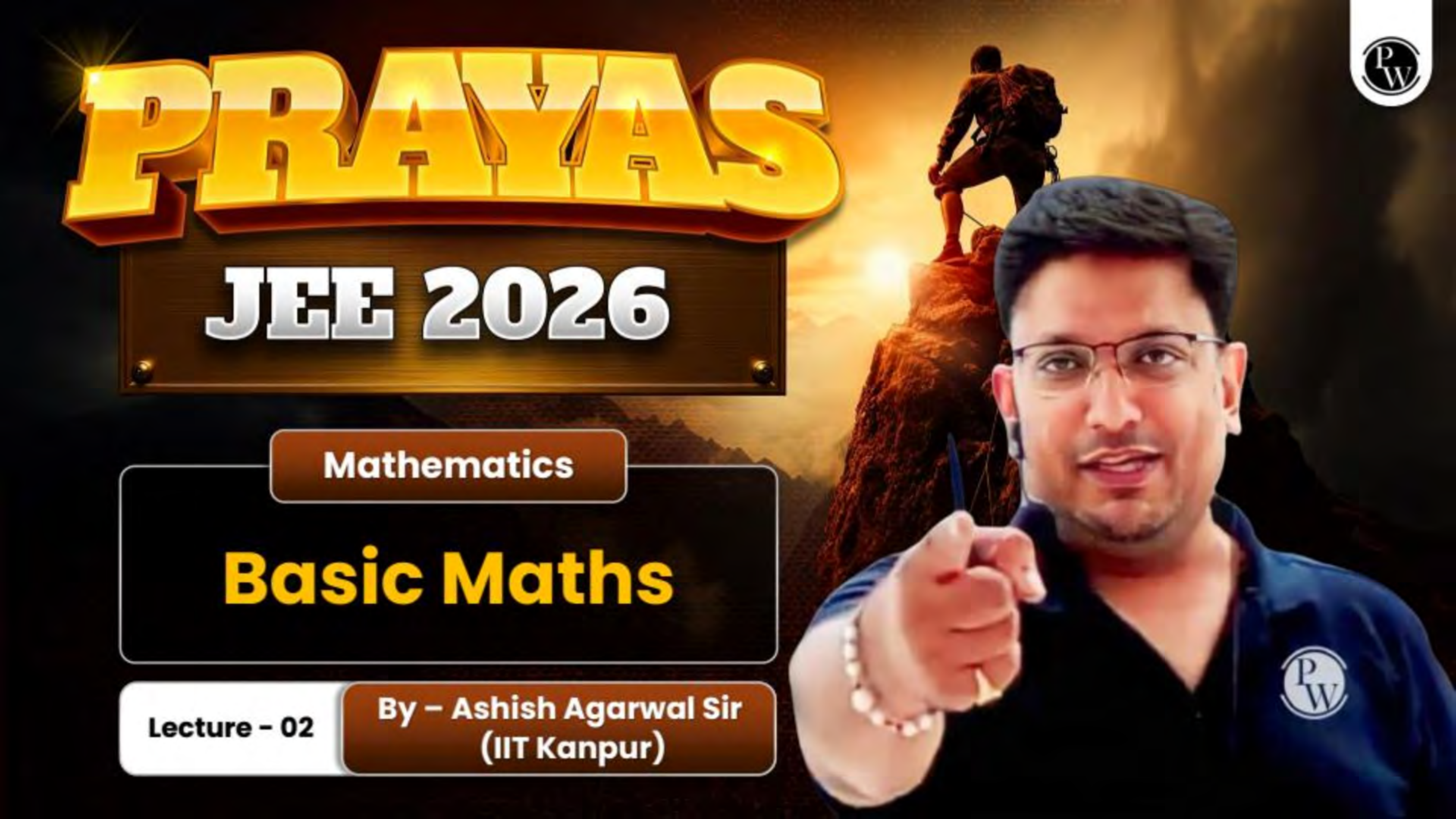

PRAYAS
JEE 2026
Mathematics
Basic Maths
Lecture - 02
By – Ashish Agarwal Sir
(IIT Kanpur)

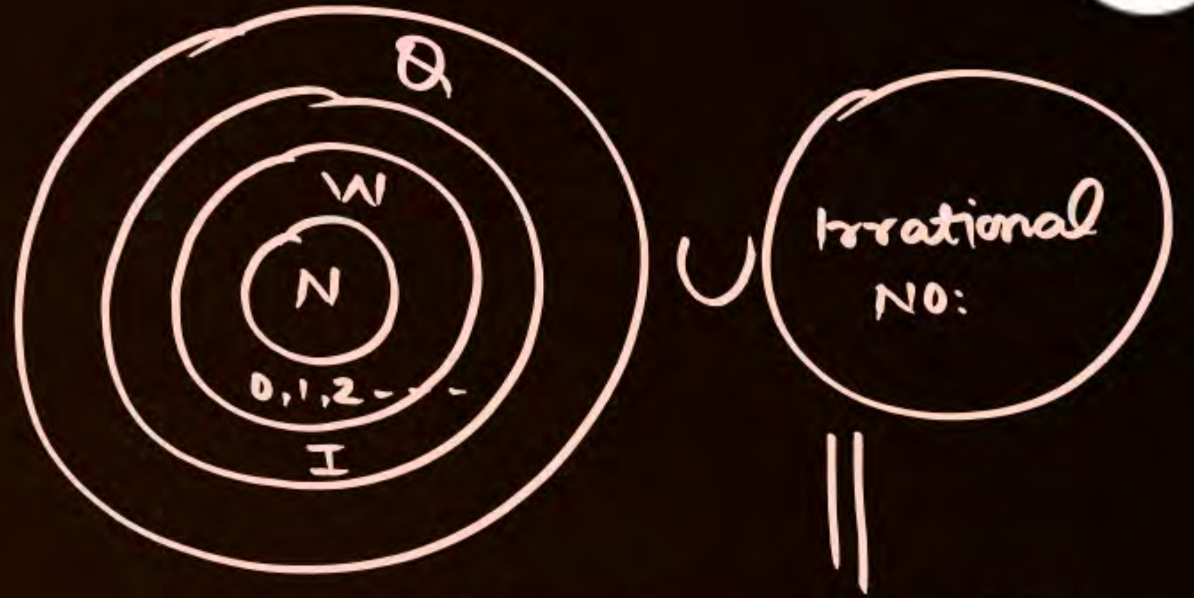
Topics *to be covered*

- **A** Important Notations
- **B** Types of Integers & Natural Numbers
- **C** Two Golden Inequalities
- **D** Some Important Points

Recap *of previous lecture*

State True or False

1. Every Natural number is an integer. (T)
2. Every integer is a rational number. (T)
3. 0 is a rational number. (T)
4. Every rational number is a terminating decimal. (F)
5. $\sqrt{2}+\sqrt{3}$ is an irrational number but not a real number. (F)
6. Every real number is a complex number. (T)
7. 5 is a natural number but not an integer. (F)
8. $\frac{343}{240}$ is a terminating decimal. (F) $\frac{343}{240} = \frac{343}{3\times 2^3 \times 2 \times 5}$



Real NO: = Rational $\cup$ Irrational

Recap *of previous lecture*

State True or False

9. $\frac{547}{2500}$ is a terminating decimal. (T)

$5^2 \times 5^2 \times 2^2 = 5^4 2^2$

10. If x^3 & x^7 are both non zero rational then x should also be rational. (T)

$x^3 \in Q$

$x^3 \cdot x^3 \in Q$

$x^6 \in Q$

Now $x^7 \in Q$

$\frac{x^7}{x^6} \in Q$

$x \in Q$

Fill in the blank

a. 

Fill in the blanks with N, I, Q.

Recap *of previous lecture*

A ke saaray elements B mai hai

A is subset of B then

* $A \cap B = A$
* $A \cup B = B$

b. Rational numbers $\cap$ irrational numbers = ϕ

c. Rational numbers $\cup$ irrational numbers = Real No:s

d. $N \cap W$ = N, $N \cup Q$ = Q

e. $I \cap Q$ = I, $I \cup Q$ = Q

f. $0.52\overline{3} = \frac{157}{300}$

$x = 0.523333\ldots$

$100x = 52.3333\ldots$ — (i)

$1000x = 523.3333\ldots$ — (ii)

(ii) − (i): $900x = 471 \Rightarrow x = \frac{471}{900} = \frac{157}{300}$

g. $\frac{\text{Any number}}{0}$ = Not defined

h. $x - \frac{2}{x+1} = 1 - \frac{2(x-1)}{x^2-1}$ then number of solutions = 0

$x - \cancel{\frac{2}{x+1}} = 1 - \cancel{\frac{2}{x+1}}$ $\quad x \neq 1$

$x = 1$

Ex: $x = 0.3\overline{4}$

$x = 0.3444\ldots$

$10x = 3.4444\ldots$

$100x = 34.444\ldots$

$90x = 31$

$x = \frac{31}{90}$

Ex: $x = 2.\overline{32}$

$x = 2.323232\ldots$

$100x = 232.323232\ldots$

$99x = 230$

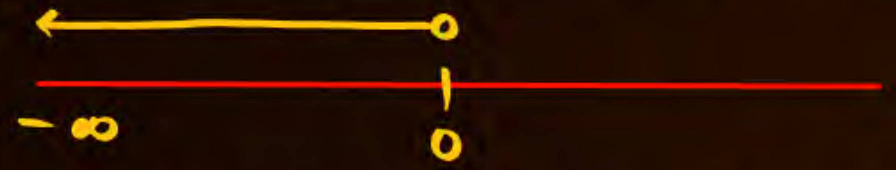
$x = \frac{230}{99}$ Ans

Important Notations

0 is neither +ve nor −ve

(i) R^+ : Set of all positive real numbers i.e. all real numbers greater than 0 i.e.
$R^+ = (0, \infty)$

(ii) R^- : Set of all negative real numbers i.e. all real numbers less than 0 i.e.
$R^- = (-\infty, 0)$



(iii) R_0 : Set all real numbers except zero, $R_0 = (-\infty, \infty) - \{0\}$.

(iv) $R^2 = R \times R =$ All points In 2-D plane

$R \times R = R^2 = \{(x, y) \mid x, y \in R\}$

(v) $R^3 = R \times R \times R =$ All points in 3-D space.

$R \times R \times R = R^3 = \{(x, y, z) \mid x, y, z \in R\}$

A close look at Set of Integers

Positive integers : $\{1, 2, 3, 4 - - -\}$

Negative integers : $\{-1, -2, -3 - - -\}$

Non Negative integers : $\{0, 1, 2, 3, 4 - - -\}$

None Positive integers : $\{0, -1, -2, -3 - - -\}$

QUESTION

Indicate which numbers in the given sets are (a) Natural numbers (b) Whole numbers (c) Integers (d) Rational numbers (e) Irrational numbers.

(i) $\left\{-10, -\sqrt{2}, -\frac{3}{4}, 0, \frac{4}{5}, \sqrt{4}, \pi, 7, \frac{18}{2}, 100\right\}$

W, N, I, Q

I, Q   R−Q   Q   I, W, Q   Q   R−Q

$7 \in I, N, W, Q$

$\frac{18}{2} \in I, N, W, Q$

$100 \in I, N, W, Q$

(ii) $\left\{-\sqrt[3]{8}, \frac{0}{3}, \sqrt[3]{7}, \sqrt{\frac{4}{9}}, 1.\overline{126}\right\}$

Tah 01(A)

* $\sqrt{4} = 2$

principal root of 4

$\sqrt{4} \notin R-Q$ Gadho/Gadhiyoo Dhyaan se Dekho

* $\sqrt{16} = 4$
* $\sqrt{9} = 3$
* $\sqrt{0} = 0$

Ex: $x^2 = 4$

$x = \pm 2$

But $\sqrt{4} = 2$

$\sqrt{a}$, $a \geq 0$ aisaa non-ve No: jiskaa square 'a' ho

QUESTION

The greatest integer lying between –10 and –15 is

(A) –10

(B) –11

(C) –15

(D) –14

Ans. B

QUESTION

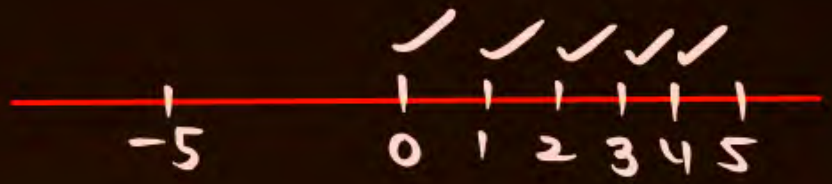
Number of whole numbers lying between –5 and 5

(A) 10

(B) 3

(C) 4

(D) 5



Any statement in Mathematics is True when it is universally true under given conditions

Any statement is false if we find an example against the statement

QUESTION

Which of the following conditions imply that the real number x is rational ?

I. $x^{1/2}$ is rational (given) $\quad x^{1/2} \in Q$ (given) $\Rightarrow x^{1/2} \cdot x^{1/2} \in Q \Rightarrow x \in Q$

II. x^2 and x^5 are rational $\quad x^2, x^5 \in Q$ (given) $\Rightarrow x^2 \cdot x^2 \in Q$ i.e $x^4 \in Q$

also $x^5 \in Q$ ✓

$\frac{x^5}{x^4} = x \in Q$, $x \neq 0$

If $x = 0 \Leftrightarrow x^2, x^5 \in Q$

III. x^2 and x^4 are rational $\quad \sqrt{2}^2, \sqrt{2}^4 = 2, 4 \in Q$

But $\sqrt{2} \notin Q$

(A) I and II only

(B) I and III only

(C) II and III only

(D) I, II, and III

Ans. A

QUESTION

Let a, b, c be positive integer S.t. $\frac{a\sqrt{2}+b}{b\sqrt{2}+c}$ is a rational number, then which of the following values of a, b, c is/are possible?

(A) $a = 1, b = 2, c = 4$

(B) $a = 4, b = 6, c = 9$

(C) $a = 2, b = 5, c = 8$

(D) $a = 3, b = 9, c = 27$

$\frac{a\sqrt{2}+b}{b\sqrt{2}+c} \in Q$ (given)

$a, b, c \in I^+$

$b^2 - ac \in I$

$b^2 - ac \neq \sqrt{2}$

$\frac{a\sqrt{2}+b}{b\sqrt{2}+c} \cdot \frac{b\sqrt{2}-c}{b\sqrt{2}-c} \in Q$

$\frac{2ab - ac\sqrt{2} + b^2\sqrt{2} - bc}{2b^2 - c^2} \in Q$

$\frac{2ab - bc + \sqrt{2}(b^2 - ac)}{2b^2 - c^2} \in Q$

$b^2 - ac = 0$

$b^2 = ac$

Types of Integers and Natural Numbers

1. Even Integers: $2n, n \in I = \{....., -6, -4, -2, 0, 2, 4, 6 .....\}$
2. Odd Integers: $2n + 1$ or $2n - 1, n \in I = \{....., -5, -3, -1, 1, 3, 5 .....\}$
3. Prime Numbers: 2, 3, 5, 7, 11, 13 ...... All natural numbers with only two divisors 1 & itself.
4. Composite Numbers: All natural numbers with more than two distinct positive divisors. Ex 4 — Divisor : 2, 4, 1
   6 → Divisors : 2, 3, 6, 1
5. Relatively Prime / Coprime: Two natural numbers whose HCF is one. Ex : 2 & 3
   8 & 21
   7 & 16
6. Twin Prime Numbers: Two prime numbers with a difference of two are called twin primes. Ex: 3 & 5, 5 & 7, 17 & 19

1 is neither prime nor Composite Number

$N - \underset{NO:S}{\text{prime}} - \{1\} = \text{Composite NO:}$

Every prime NO: ≥ 5 is of type $6K \pm 1$, $K \in N$

$5 = 6 \cdot 1 - 1$

$7 = 6 \cdot 1 + 1$

$11 = 6 \cdot 2 - 1$

$13 = 6 \cdot 2 + 1$

$17 = 6 \cdot 3 - 1$

$23 = 6 \cdot 4 - 1$

$19 = 6 \cdot 3 + 1$

But Every NO: of form $6K \pm 1$ is not prime

$6 \cdot 4 + 1 = 25 \notin$ prime

$6 \cdot 6 - 1 = 35 \notin$ prime.

Kaam ki Baatien

composite
1, 2, 3, 4, 5, 6, 7, 8 - - -
prime composite

1. 2 is smallest prime.

2. 4 is the smallest composite number.

3. 1 is neither prime nor composite.

4. Every prime ≥ 5 is of type ________ where ________ but every number of form ________ is not ________.

QUESTION

If 'a' and 'b' are two distinct prime numbers lying between 1 and 10, which of the following can be the sum of 'a' and 'b' -

7 12
2, 3, 5, 7
5 8 10

(A) 5

(B) 6

(C) 7

(D) 8

Ans. A, C, D

Two Golden Inequalities

$G_1 : x + \frac{1}{x} \geq 2, \quad x \in R^+$

$G_2 : x + \frac{1}{x} \leq -2, \quad x \in R^-$

★ any +ve Real + its reciprocal ≥ 2

★ any -ve Real + its reciprocal ≤ -2

$x + \frac{1}{x} = 2,\ x \in R^+ \iff x = 1$

$x + \frac{1}{x} = -2,\ x \in R^- \iff x = -1$

proof: since $x \in R^+ \Rightarrow \sqrt{x} \in R^+$

(i) $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \geq 0$ $\quad \frac{1}{\sqrt{x}} \in R^+$

$x + \frac{1}{x} - 2\sqrt{x} \cdot \frac{1}{\sqrt{x}} \geq 0$

$x + 1/x \geq 2$. (Hence proved)

If $x \in R$ then $x^2 \geq 0$

Ex: $(-\sqrt{2})^2 = 2$

$(-5)^2 = 25$

$(3)^2 = 9$

$0^2 = 0$

(ii) since $x \in R^-$ let $x = -t$, $t > 0 \Rightarrow t + \frac{1}{t} \geq 2$ from (i)

$\Downarrow$

$t = -x$

$-x - \frac{1}{x} \geq 2 \Rightarrow x + 1/x \leq -2$

$f(x) = x + \frac{1}{x}$ Range : $(-\infty, -2] \cup [2, \infty)$

$f(x) = k$ has a real soln only if $k \in$ Range of $f(x)$

Ex: $\sin x = 2 \longrightarrow$ No soln

Ex: $\cos x = -2 \longrightarrow$ No soln.

QUESTION [JEE Mains 2023 (24 Jan)]

The number of real solutions of the equation $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$,is

(A) 3

(B) 4

(C) 0

(D) 2

$x+\frac{1}{x}=t$

$\underbrace{S.B.S}$ $x^2+\frac{1}{x^2}+2x\cdot\frac{1}{x}=t^2$

$x^2+\frac{1}{x^2}=t^2-2$

$3(t^2-2)-2t+5=0$

$3t^2-2t-1=0$

$3t^2-3t+t-1=0$

$(3t+1)(t-1)=0$

$t=-1/3, 1$

$x+1/x=-1/3$ or $x+1/x=1$ No real soln $1 \notin$ Range of $x+1/x$

b'coz $-1/3$ does not lie in Range of $x+1/x$ No real soln

Ans. C

QUESTION

Consider the expressions

$E_1 = x + \frac{1}{x}, x \in R^-$; $E_2 = y^2 + \frac{1}{y^2}, y \in R_0$; $E_3 = z^2 + \frac{1}{z^2+1} + \overset{1+1}{2}, z \in R$

If $\alpha = \max E_1, \beta = \min E_2, \gamma = \min E_3$ then $\alpha + \beta + \gamma$ is

(A) 3

(B) 5

(C) 2

(D) 7

$E_1 = x + \frac{1}{x}$ $x \in R^- \Rightarrow E_1 = x + \frac{1}{x} \leq -2$, $E_2 = y^2 + \frac{1}{y^2}$, $y \in R_0 \Rightarrow$ clearly $y^2 > 0$ i.e $y^2 \in R^+$

$\alpha = \max E_1 = -2$ @ $x = -1$

$\Rightarrow E_2 = y^2 + \frac{1}{y^2} \geq 2$

$\beta = \min E_2 = 2$ @ $y^2 = 1 \Rightarrow y = \pm 1$

$E_3 = z^2 + 1 + \frac{1}{z^2+1} + 1, z \in R$

$z^2 \geq 0 \Rightarrow 1 + z^2 \geq 1$

$1 + z^2 \in R^+$

$(z^2+1) + \frac{1}{z^2+1} \geq 2$

$(z^2+1) + \frac{1}{z^2+1} + 1 \geq 3$

$\gamma = \min E_3 = 3$ @ $z^2 + 1 = 1 \Rightarrow z = 0$

$\alpha + \beta + \gamma = -2 + 2 + 3 = 3$.

Ans. A

QUESTION

Tanoz

If a, b, c are non-zero real numbers, then the minimum value of expression

$$\left(\frac{(a^4+3a^2+1)(b^4+5b^2+1)(c^4+7c^2+1)}{a^2b^2c^2}\right) \text{ is}$$

Ans. 315

Yaad Karnay hai

$\sqrt{1} = 1$	$\sqrt{11} = 3.3166$
$\sqrt{2} = 1.4142$	$\sqrt{12} = 3.4641$
$\sqrt{3} = 1.732$	$\sqrt{13} = 3.6055$
$\sqrt{4} = 2$	$\sqrt{14} = 3.7416$
$\sqrt{5} = 2.236$	$\sqrt{15} = 3.8729$
$\sqrt{6} = 2.4494$	$\sqrt{16} = 4$
$\sqrt{7} = 2.6457$	$\sqrt{17} = 4.1231$
$\sqrt{8} = 2.8284$	$\sqrt{18} = 4.2426$
$\sqrt{9} = 3$	$\sqrt{19} = 4.3588$
$\sqrt{10} = 3.1622$	$\sqrt{20} = 4.4721$

Yaad Karnay hai

$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$6^2 = 36$	$7^2 = 49$	$8^2 = 64$	$9^2 = 81$	$10^2 = 100$
$11^2 = 121$	$12^2 = 144$	$13^2 = 169$	$14^2 = 196$	$15^2 = 225$
$16^2 = 256$	$17^2 = 289$	$18^2 = 324$	$19^2 = 361$	$20^2 = 400$
$21^2 = 441$	$22^2 = 484$	$23^2 = 529$	$24^2 = 576$	$25^2 = 625$
$26^2 = 676$	$27^2 = 729$	$28^2 = 784$	$29^2 = 841$	$30^2 = 900$

Yaad Karnay hai

$31^2 = 961$	$32^2 = 1024$	$33^2 = 1089$	$34^2 = 1156$	$35^2 = 1225$
$36^2 = 1296$	$37^2 = 1369$	$38^2 = 1444$	$39^2 = 1521$	$40^2 = 1600$
$41^2 = 1681$	$42^2 = 1764$	$43^2 = 1849$	$44^2 = 1936$	$45^2 = 2025$
$46^2 = 2116$	$47^2 = 2209$	$48^2 = 2304$	$49^2 = 2401$	$50^2 = 2500$
$51^2 = 2601$	$52^2 = 2704$	$53^2 = 2809$	$54^2 = 2916$	$55^2 = 3025$
$56^2 = 3136$	$57^2 = 3249$	$58^2 = 3364$	$59^2 = 3481$	$60^2 = 3600$

Don’t Forget to
Retry all the class illustrations

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

KTK 1

The equation $\frac{2x^2}{x-1} - \frac{2x+7}{3} + \frac{4-6x}{x-1} + 1 = 0$ has the roots-

(A) 4 and 1

(B) only 1

(C) only 4

(D) neither 4 nor 1

Ans. C

QUESTION

KTK 2

Which one of the following does not reduce to $\sin x$ for every x, wherever defined, is

A $\dfrac{\tan x}{\sec x}$

B $\dfrac{\sin x}{\sec^2 x - \tan^2 x}$

C $\dfrac{\sin^2 x \sec x}{\tan x}$

D All reduce to $\sin x$

Ans. D

QUESTION

KTK 3

	Column-I		Column-II
(A)	A rectangular box has volume 48, and the sum of the length of the twelve edges of the box is 48. The largest integer that could be the length of an edge of the box, is	(P)	1
(B)	The number of zeroes at the end in the product of first 20 prime numbers, is	(Q)	2
(C)	The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is	(R)	3
		(S)	4
		(T)	6

Ans. (A) T; (B) P; (C) P

QUESTION

KTK 4

The ratio of total area of the rectangle to the total shaded area

(A) $\frac{2}{\pi}$

(B) $\frac{4}{4-\pi}$

(C) $\frac{4-\pi}{\pi}$

(D) $\frac{\pi}{4}$

Ans. B

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